

# A Level Further Mathematics

## Introduction and Summer Work



### Introduction

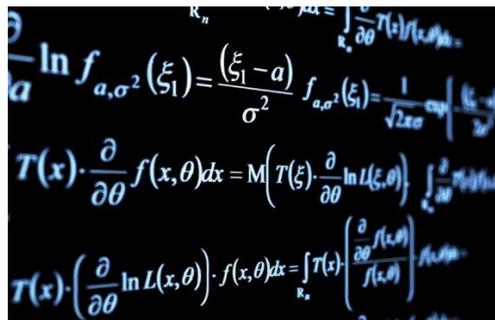
Remember that A level Further Mathematics is an additional A level to Maths. You need to bring the same equipment to lessons, although you'll need to have a separate folder for each subject. You must score highly on the Maths Skills Assessment as well as complete the following summer work to be suitable for this course.

### Summer Work

The following questions are challenging, but are based on GCSE knowledge; you should know everything you need to know already, but they will require some problem solving skills.

What I am looking for is how well you can communicate mathematically. A list of correct answers will score you some marks, but **well explained, coherent** and **concise** solutions will score you many more marks.

You must write your solutions neatly onto lined (or squared) A4 paper and bring them to your first Further Maths lesson in September.



If you have any questions please contact me. Best of luck with your exam results and I look forward to welcoming you in September.

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### Section 1 – Algebraic reasoning

1. Expand and simplify  $(x + y)^4$
2. Write  $4x^4 - 13x^2 + 9$  as the product of four linear factors
3. (a) Simplify  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$   
(b) Hence show that

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{24} + \sqrt{25}} = 4$$

4. Given that  $x$  is positive, solve the equation

$$\frac{1}{x} + \frac{1}{x + 2} = \frac{28}{195}$$

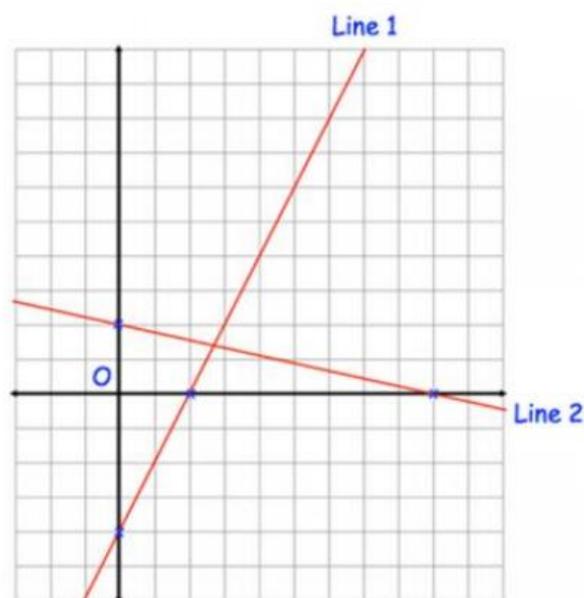
5. A straight line passes through  $(7a, 5)$  and  $(3a, 3)$ .  
An equation of this line is  $x + by - 12 = 0$ . Find the values of  $a$  and  $b$ .

6. Ben is using the quadratic formula to solve an equation of the form  $ax^2 + bx + c = 0$ .  
He has got to this point in his working out:

$$x = \frac{3 \pm \sqrt{29}}{2}$$

Find the values of  $a$ ,  $b$  and  $c$  for the equation he is solving.

7. Shown are two straight lines drawn on the grid. Line 1 has equation  $y = 3x - 12$ .  
(a) Find the equation of line 2.  
(b) Are the lines perpendicular? Explain your answer.



8. The graph of  $y = ax^2 + bx + c$  has a vertex at  $(5, -3)$  and the curve passes through  $(4, 0)$ . Find the values of  $a$ ,  $b$  and  $c$ .
9. A swimmer dives into a pool. Her position,  $p$  metres, underwater can be modelled in relation to her horizontal distance,  $x$  metres, from the point she entered the water as a quadratic equation  $p = \frac{1}{2}x^2 - 3x$ . The position of the bottom of the pool can be modelled by the linear equation  $p = 0.3x - 6$ . Determine whether this model predicts that the swimmer will touch the bottom of the pool.

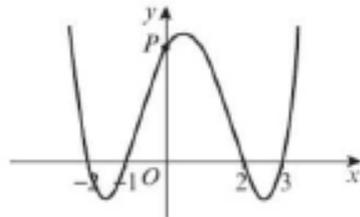
10. (a) Show that the solutions to the equation  $ax^2 + 2bx + c = 0$  are given by

$$x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - ac}{a^2}}$$

- (b) Hence show that the solutions to the equation  $ax^2 + 2bx + c = 0$  can be written as

$$x = \frac{-b \pm \sqrt{b^2 - ac}}{a}$$

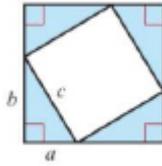
11. The graph of  $y = x^4 + ax^3 + bx^2 + cx + d$  is shown, where  $a$ ,  $b$ ,  $c$  and  $d$  are real constant. The curve crosses the  $y$ -axis at point  $P$ .



- (a) Find the coordinates of point  $P$
- (b) Find the values of  $a$ ,  $b$ ,  $c$  and  $d$ .
12. Accident investigators are studying the stopping distance of a particular car.
- When the car is travelling at 20mph, its stopping distance is 6 feet.
- When the car is travelling at 30mph, its stopping distance is 14 feet.
- When the car is travelling at 40mph, its stopping distance is 24 feet.
- The investigators suggest that the stopping distance,  $D$  (in feet) can be modelled by a quadratic function in terms of speed,  $s$  (in miles per hour), as
- $$D(s) = as^2 + bs + c$$
- (a) Find the values of the constants  $a$ ,  $b$  and  $c$ .
- (b) At an accident scene a car has left behind a skid that is 20 feet long. Use the model to calculate the speed that this car was going before the accident.

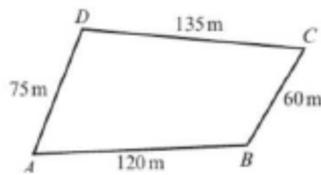
## Section 2 – Geometrical reasoning

1. The diagram shows a square and four congruent right-angled triangles.

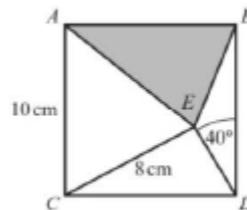


Use the diagram to prove that  $a^2 + b^2 = c^2$ .

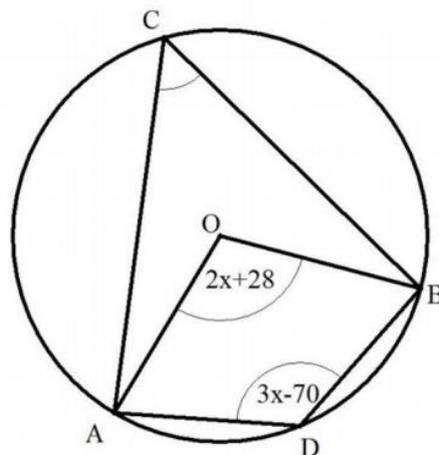
2. A farmer has a field in the shape of a quadrilateral as shown. The angle between fences  $AB$  and  $AD$  is  $74^\circ$ . Find the angle between fences  $BC$  and  $CD$ .



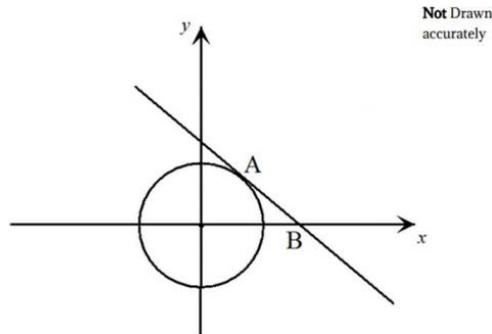
3.  $ABCD$  is a square. Angle  $CED$  is obtuse. Find the area of the shaded triangle.



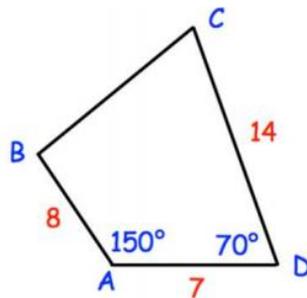
4. Points  $A$ ,  $B$ ,  $C$  and  $D$  are all on the circumference of the circle.  $O$  represents the centre. Angles  $AOB$  and  $ADB$  are expressed in terms of  $x$ . Calculate the value of  $x$ , giving reason for each stage of your working.



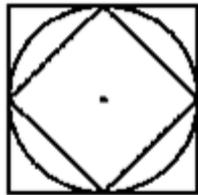
5. Consider the circle with equation  $x^2 + y^2 = 13$  sketched below. The point  $A$  lies on the circle and has a  $y$ -ordinate of 2. The tangent to the circle at  $A$  intersects the  $x$ -axis at the point  $B$ . Find the coordinates of  $B$ .



6. In quadrilateral  $ABCD$ , various lengths and angles have been labelled. Calculate the length of  $BC$ . Given your answer correct to 3 significant figures.



7. The diagram shows two squares and a circle. Given that  $\pi$  is defined as the circumference of a circle of diameter 1 unit, prove that  $2\sqrt{2} < \pi < 4$ .



8. The diagram shows 3 identical circles inside a rectangle. Each circle touches the other two circles and the sides of the rectangle, as shown in the diagram. Given that the radius of each circle is 24mm, work out the area of the rectangle. Give your answer correct to 3 significant figures.

